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1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$m^2 + 4m + 5y = 0$

Using $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a=1, b=4, c=5$

$\therefore \frac{-4 \pm \sqrt{4^2 - (4\times1\times5)}}{2\times1}$

$= \frac{-4 \pm \sqrt{16-20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$m = -2 \pm j \quad m_1 = -2+j, \; m_2 = -2-j$

$\therefore y = e^{-2}[A\cos j\theta + B\sin j\theta)$

$y = C\cos\theta + D\sin\theta$

$\frac{dy}{dx} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{dx^2} = -C\cos\theta - D\sin\theta$

$\therefore -(C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$(-C + 4D + 5C)\cos\theta + (-D - 4C + 5D)\sin\theta = 6\sin\theta$

$(-C + 4D + 5C) = 0$

$-4C + 4D = 0$

$(-D - 4C + 5D) = 6$

$\therefore -4C + 5D = 6$

$\therefore -4C + 4D = 0 \quad ---(b)$

$-4C + 4D = 6 \quad ---(a)$

(a) + (b)

$-4C + 4D = 6$

$+4C + 4D = 0$

$0 \quad 8D = 6$

$\therefore D = 6/8 = 3/4$

Putting $D = 6$ in equ (b)

$-4C + 4(3/4) = 0$

$4C = 3$

$C = 3/4$

$\therefore y = e^{-2\theta}[A\cos\theta + B\sin\theta] + 3/4\cos\theta + 3/4\sin\theta$

$\therefore y = e^{-2\theta}(A\cos\theta + B\sin\theta] + 0.75(-\cos\theta + \sin\theta)$

where $G.S = C.I + P.I$

ii) Eliminating C.I

$y = 0.75(-\cos\theta + \sin\theta)$

$\therefore y = -0.75(\cos\theta - \sin\theta$

from $0^\circ + 270^\circ$

iii) Therefore the steady state value =

2) $EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

$EI\frac{d^2y}{dx^2} = 0$

$EIm^2 = 0$

$\therefore m^2 = 0/EI$

$m = \sqrt{0}$

$\therefore y = e^{0x}(A+Bx)$

CI: $y = (A+Bx)$

$\therefore PI: y = Cx^4 + dx^3 + ex^2$

$\frac{dy}{dx} = 4Cx^3 + 3dx^2 + 2ex$

$\frac{d^2y}{dx^2} = 12Cx^2 + 6dx + 2e$

$\therefore EI[12Cx^2 + 6dx + 2e] = \frac{w}{2}(L^2 - 2Lx + Dx^2)$

$\therefore$ Comparing the coefficients

$EI\,2e = \frac{wL^2}{2}$

$\therefore e = \frac{wL^2}{4EI}$

$EI\,6d = \frac{-wL}{2}$

$d = \frac{-wL}{6EI}$

$EI\,12Cx^2 = \frac{wx^2}{2}$

$C = \frac{w}{24EI}$

$$\therefore P.I = \frac{w}{24EI}x^4 + \frac{wl}{6EI}x^3 + \frac{wl^2}{4EI}x^2$$

$$= \frac{wx^4 - 4wlx^3 + 6wl^2x^2}{24EI}$$

$$= \frac{w}{24EI}\left[x^4 - 4lx^3 + 6l^2w^2\right]$$

Since G.S = C.F + P.I

$$G.S: y = (A + Bx) + \frac{w}{24EI}\left(x^4 - 4lx^3 + 6l^2w^2\right)$$

at $y = 0 \quad x = 0$

$$0 = A + 0 + \frac{w}{24EI}(0 - 0 + 0)$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}(4x^3 - 12lx^2 + 12l^2x)$$

at $\frac{dy}{dx} = 0, \quad x = 0$

$$0 = B + \frac{w}{24EI}(0)$$

$$\therefore B = 0$$

$$y = (0 + 0x) + \frac{w}{24EI}(x^4 - 4lx^3 + 6l^2x^2)$$

$$y = \frac{w}{24EI}(x^4 - 4lx^3 + 6l^2x^2)$$

∴ Assuming $x = l$

$$y = \frac{w}{24EI}\left[l^4 - 4l \times l^3 + 6l^2 \cdot l^2\right)$$

$$y = \frac{w}{24EI}\left[l^4 - 4l^4 + 6l^4\right)$$

$$y = \frac{w}{24EI}\left[3l^4\right]$$

$$y = \frac{3WL^4}{24EI}$$

$$y = \frac{WL^4}{8EI}$$